

A model of the electroweak interaction

E.K. Loginov*

Department of Physics, Ivanovo State University, Ermaka St. 39, Ivanovo, 153025, Russia

(Dated: November 16, 2010)

We consider a modification of the standard electroweak model with one family of fermions. This model has only five free parameters: the gauge coupling g , the scalar self-coupling μ^2 , and the three Yukawa couplings. In particular, the model predicts a value of the Higgs boson mass.

The standard electroweak model [1] is a mathematically consistent renormalizable field theory which predicts or is consistent with all experimental facts. It successfully predicted the existence and form of the weak neutral current, the existence and masses of the W and Z bosons, and the charm quark, as necessitated by the GIM mechanism. The charged current weak interactions, as described by the generalized Fermi theory, were successfully incorporated, as was quantum electrodynamics. The consistency between theory and experiment indirectly tested the radiative corrections and ideas of renormalization and allowed the successful prediction of the top quark mass. However, despite the apparent striking success of the theory, there are a lot of reasons why it is not the ultimate theory. In particular, the theory has far too much arbitrariness to be the final story.

In fact the standard theory, even with only one family of fermions, has seven arbitrary parameters: the two gauge couplings g and g' , the two scalar self-couplings μ^2 and λ , and the three Yukawa couplings f_e , f_u , and f_d (if the right-handed neutrinos). We note that the electroweak unification is in a sense not complete: we need to insert two gauge coupling constants g and g' to account for these two classes of interactions. These and other deficiencies of the standard electroweak model motivated the effort to construct theories with higher unification of gauge symmetries. However, even if the standard model group is embedded into a simple gauge group, the above-mentioned arbitrariness is not much improved. Furthermore, there is the acute gauge hierarchy problem.

In this paper we will introduce a noncompact simple group which contains the $SU(2) \times U(1)_Y$ group of the standard electroweak model as a subgroup. At the same time the above problems will motivate us to investigate the possibility of the spontaneous symmetry breaking only at the energy scale M_0 that fixed by the weak interaction strength. We will construct a real model of the electroweak interaction, the number of free parameters of which are smaller than in the standard model. In particular, we will show that the model predicts a value of the Higgs boson mass.

Consider the standard electroweak model with its fermionic sector composed of e , ν_e leptons only. With the fermions the gauge-invariant Lagrangian takes on the

form

$$\mathcal{L}_\psi^l = \bar{\psi}_L i \gamma^\mu D_\mu \psi_L + \bar{\psi}_R^\sigma i \gamma^\mu D_\mu \psi_R^\sigma - \left(f_\nu \bar{\psi}_L \tilde{\phi} \nu_R + f_e \bar{\psi}_L \phi e_R + h.c. \right), \quad (1)$$

where left-handed leptons $\psi_L = (\nu_L, e_L)$ transform as $SU(2)$ doublet, while the right-handed fields ν_R and e_R are singlets. We have tentatively included $SU(2)$ -singlet right-handed neutrinos ν_R in (1), because they are required in many models for neutrino mass. However, they are not necessary for the consistency of the theory. We rewrite the Lagrangian (1) in a slightly different way. We first combine the right-handed fermions into the doublet $\psi_R = (\nu_R, e_R)$ but not $SU(2)$ doublet and define the covariant derivative D_μ and the operator M by

$$D_\mu \psi = \left(\partial_\mu - i \gamma_L \frac{g}{2} A_\mu^k \sigma_k - i \frac{g_1}{2} Y B_\mu \right) \psi, \quad (2)$$

$$M \psi = (\gamma_R \Phi + \gamma_L \Phi^\dagger) \psi, \quad (3)$$

where $\psi = \psi_L + \psi_R$, the matrix $\Phi = (f_\nu \tilde{\phi}, f_e \phi)$, and the projection operators $\gamma_{L,R} = \frac{1}{2}(1 \pm \gamma_5)$. As usual the hypercharge values are defined as twice the average charges of each multiplet, i.e. $Y(\psi_L) = -1$, $Y(\nu_R) = 0$ and $Y(e_R) = -2$. Using these notations, we rewrite the Lagrangian (1) as

$$\mathcal{L}_\psi^l = \bar{\psi} (i \gamma^\mu D_\mu - M) \psi. \quad (4)$$

Further, we combine the left- and right-handed fermions in the quartet $\Psi = (\psi_L, \psi_R)$ and rewrite the covariant derivative (2) in the form

$$D_\mu \Psi = \begin{pmatrix} D_\mu^L & 0 \\ 0 & D_\mu^R \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad (5)$$

with the operators

$$D_\mu^L = \partial_\mu - i \gamma_L \frac{g}{2} A_\mu^k \sigma_k - i \frac{g'}{2} Y_L B_\mu, \quad (6)$$

$$D_\mu^R = \partial_\mu - i \frac{g'}{2} Y_R B_\mu, \quad (7)$$

where Y_L and Y_R are scalar 2×2 matrices with the diagonal elements $-1, -1$ and $0, -2$ respectively. Similarly, we rewrite the expression (3) as

$$M \Psi = \begin{pmatrix} 0 & \gamma_R \Phi \\ \gamma_L \Phi^\dagger & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}. \quad (8)$$

Using these notations, we write the Lagrangian (4) in the form

$$\mathcal{L}_\psi^l = \bar{\Psi} (i\gamma^\mu D_\mu - M) \Psi. \quad (9)$$

In the same way, we can rewrite the Lagrangian $\mathcal{L}_\psi^{q\alpha}$ including the quarks of one family. Thus, the complete gauge-invariant Lagrangian including fermions of the first family is

$$\mathcal{L}_\psi = \mathcal{L}_\psi^l + \sum_\alpha \mathcal{L}_\psi^{q\alpha}, \quad (10)$$

where $\mathcal{L}_\psi^{q\alpha}$ is the Lagrangian of the form (9) and α is the colour index of the quark fields.

Now we turn to the construction of the gauge-invariant Lagrangian including only vector and scalar fields. To do this, we redefine the operator M . Suppose

$$M = \begin{pmatrix} 0 & \gamma_R \Phi \\ \gamma_L \Phi^\dagger & i\mu \end{pmatrix}, \quad \Phi = f_\eta(\tilde{\phi}, \phi), \quad (11)$$

where μ and f_η are real constants. To obtain the anti-symmetric second-rank tensor we follow the Yang-Mills construction and study the combination

$$\tilde{F}_{\mu\nu} = [\tilde{D}_\mu, \tilde{D}_\nu], \quad \tilde{D}_\mu = ID_\mu + \frac{1}{4}\gamma_\mu M, \quad (12)$$

where I is identity matrix in the Dirac algebra and D_μ is the covariant derivative defined in (5). Further, we consider the Lagrangian

$$\mathcal{L}_F^l = \frac{1}{4g^2} \text{tr} \left(\tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} \right). \quad (13)$$

Using the standard properties of the gamma matrices and the identity $\text{tr}(\Phi\Phi^\dagger) = \text{tr}(\Phi^\dagger\Phi)$, we find

$$\begin{aligned} \mathcal{L}_F^l &= \frac{1}{4g^2} \text{tr} \left\{ 4[D_\mu, D_\nu]^2 \right. \\ &\quad \left. + \frac{3}{2}(D_\mu\Phi)^\dagger(D^\mu\Phi) + \frac{3}{2}\mu^2\Phi^\dagger\Phi - \frac{3}{4}(\Phi^\dagger\Phi)^2 \right\}, \end{aligned} \quad (14)$$

where the covariant derivative

$$D_\mu\Phi = \left(\partial_\mu - i\frac{g}{2}A_\mu^k\sigma_k \right) \Phi - i\frac{g'}{2}(Y_L\Phi - \Phi Y_R)B_\mu. \quad (15)$$

We now substitute Φ determined in (11) into (14). Using the equality $(D_\mu\tilde{\phi})^\dagger(D^\mu\tilde{\phi}) = (D_\mu\phi)^\dagger(D^\mu\phi)$ and noting that $\text{tr}(Y_L^2 + Y_R^2) = 6$, we get

$$\begin{aligned} \mathcal{L}_F^l &= -\frac{1}{4}F_{\mu\nu}^k F^{k\mu\nu} - \frac{3g'^2}{2g^2} G_{\mu\nu} G^{\mu\nu} \\ &\quad + \frac{3f_\eta^2}{4g^2} \left[(D_\mu\phi)^\dagger(D^\mu\phi) + \mu^2\phi^\dagger\phi - \frac{f_\eta^2}{2}(\phi^\dagger\phi)^2 \right], \end{aligned} \quad (16)$$

where the Yang-Mills field strength and the covariant derivative of the scalar field are the usual form

$$\begin{aligned} F_{\mu\nu}^k &= \partial_\mu A_\nu^k - \partial_\nu A_\mu^k + g\varepsilon_{ijk}A_\mu^i A_\nu^j, \\ G_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\ D_\mu\phi &= \left(\partial_\mu - i\frac{g}{2}A_\mu^k\sigma_k - i\frac{g'}{2}B_\mu \right) \phi. \end{aligned} \quad (17)$$

In the same way, we can construct the Lagrangian $\mathcal{L}_F^{q\alpha}$ which will differ from (16) only by $\text{tr}(Y_L^2 + Y_R^2) = 22/9$. Thus, if we define

$$\mathcal{L}_F = \frac{1}{4} \left(\mathcal{L}_F^l + \sum_\alpha \mathcal{L}_F^{q\alpha} \right), \quad f_\eta^2 = \frac{4}{3}g^2, \quad (18)$$

then the gauge-invariant Lagrangian including the vector and scalar fields take the following form

$$\begin{aligned} \mathcal{L}_F &= -\frac{1}{4}F_{\mu\nu}^i F^{i\mu\nu} - \frac{1}{4} \left(\frac{10g'^2}{3g^2} \right) G_{\mu\nu} G^{\mu\nu} \\ &\quad + \left[(D_\mu\phi)^\dagger(D^\mu\phi) + \mu^2\phi^\dagger\phi - \frac{2g^2}{3}(\phi^\dagger\phi)^2 \right]. \end{aligned} \quad (19)$$

Now we note that the operator \tilde{D}_μ in (12) may be represented as $\tilde{D}_\mu = \partial_\mu + \tilde{A}_\mu$, where \tilde{A}_μ is 16×16 matrices for leptons and traceless 64×64 matrices for leptons and quarks. Let L_0 be a simple Lie algebra containing all these traceless matrices and G_0 be its Lie group containing the $SU(2) \times U(1)_Y$ group as a subgroup. Suppose that the spontaneous symmetry breaking takes place in one stages, characterized by the energy scale M_0 . It characterizes the spontaneous symmetry breakings of both G_0 to a subgroup H_0 and $SU(2) \times U(1)_Y$ to $U(1)_{em}$ simultaneously. We must prove that M_0 fixed by the weak interaction strength. To do this, we note first that the equality

$$g' = \sqrt{\frac{3}{10}}g \quad (20)$$

is valid in the G_0 limit; i.e. for the energy scale $M \geq M_0$. Now we study the regime $M < M_0$. The evolution of couplings in gauge theories with the groups $SU(n)$ is described by the renormalization group equation

$$\frac{dg_n}{d(\ln M)} = -b_n g_n^3, \quad (21)$$

where the difference

$$b_n - b_1 = 11n/48\pi^2, \quad n \geq 2. \quad (22)$$

We have ignored the contribution coming from the Higgs scalar. The solution of the renormalization group equation is

$$g_n^{-2}(M) = g_n^{-2}(M_0) - 2b_n \ln \frac{M_0}{M}. \quad (23)$$

We can express the low-energy couplings g_1 and g_2 in terms of more familiar parameters by using

$$g_1^2 = g_2^2 = g^2 = 4\pi\alpha_0, \quad (24)$$

at the energy scale $M = M_0$ and

$$g_2^2 \sin^2 \theta_W = \frac{3}{10} g_1^2 \cos^2 \theta_W = 4\pi\alpha(M) \quad (25)$$

at the energy scale $M < M_0$. Subtracting these equalities into (23) and using (22) for $n = 2$, we get

$$\sin^2 \theta_W = \frac{3}{13} - \frac{110\alpha(M)}{39\pi} \ln \frac{M_0}{M}. \quad (26)$$

This determines the unification scale M_0 if we identify M with the mass M_W of the gauge boson W_μ^\pm and use the mass relation

$$\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2}. \quad (27)$$

Given the Standard Model inputs [2]:

$$\alpha^{-1}(M_W) = 128.267 \pm 0.025, \quad (28)$$

$$M_W = 80.3840 \pm 0.0140 \text{ GeV}, \quad (29)$$

$$M_Z = 91.1876 \pm 0.0021 \text{ GeV}, \quad (30)$$

we have

$$M_0 = 247 \pm 11 \text{ GeV}. \quad (31)$$

It follows from here that we can indeed identify M_0 with the vacuum expectation value in the electroweak theory, i.e.

$$M_0 = \left(\sqrt{2} G_F \right)^{-1/2} = 246,2209 \pm 0.0005 \text{ GeV}. \quad (32)$$

Again we consider the Lagrangian (19). Obviously, it coincides with the standard electroweak model Lagrangian for the energy scale M_0 as given by the vacuum expectation value of the Higgs multiplet. However, in

contrast to the standard model, the scalar self-coupling in this model is defined by

$$\lambda = \frac{2g^2}{3}. \quad (33)$$

Therefore in the model the mass of the physical Higgs scalar particle is given by

$$M_H = \frac{4\sqrt{3}}{3} M_W. \quad (34)$$

Finally, substituting (28), (30), and (32) into (26) and (27), we obtain

$$M_W = 80,3829 \pm 0.0019 \text{ GeV}, \quad (35)$$

$$M_H = 185,6362 \pm 0.0042 \text{ GeV}. \quad (36)$$

Note that the current lower limit on the standard model Higgs boson mass from LEP experiments is $M_H > 114.4$ GeV at 95% C.L. [3] Analysis of high-precision measurements of electroweak observables leads to indirect upper bound [4] $M_H < 186$ GeV at 95% C.L. on the Higgs boson mass. Thus, the obtained value of the Higgs boson mass in fact coincides with the upper bound of the standard model Higgs boson mass. Note also that the gauge couplings g' is determined by the value of the gauge couplings g . This statement follows immediately from (26) and (27).

* Electronic address: ek.loginov@mail.ru

- [1] S.L. Glashow, Nucl. Phys. **22**, 579 (1961); S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967); A. Salam, In Elementary Particle Theory (ed. N. Svartholm), Almqvist and Wilsell, Stockholm, 1968.
- [2] K. Nakamura et al., J. Phys. **G37**, 075021 (2010).
- [3] The LEP working group for Higgs boson searches, CERN-EP **055** (2001).
- [4] M.W. Grunewald, Nucl. Phys. Proc. Suppl. **B117**, 280 (2003).